PROBLEMS

1, 2, 3 = straightforward, intermediate, challenging □ = full solution available in the *Student Solutions Manual and Study Guide*

= coached solution with hints available at http://www.pse.com = computer useful in solving problem

= paired numerical and symbolic problems

Section 31.1 Faraday's Law of Induction
Section 31.3 Lenz's Law

1. A 50-turn rectangular coil of dimensions 5.00 cm × 10.0 cm is allowed to fall from a position where $B = 0$ to a new position where $B = 0.500$ T and the magnetic field is directed perpendicular to the plane of the coil. Calculate the magnitude of the average emf that is induced in the coil if the displacement occurs in 0.250 s.

2. A flat loop of wire consisting of a single turn of cross-sectional area 8.00 cm^2 is perpendicular to a magnetic field that increases uniformly in magnitude from 0.500 T to 2.50 T in 1.00 s. What is the resulting induced current if the loop has a resistance of 2.00 Ω?

3. A 25-turn circular coil of wire has diameter 1.00 m. It is placed with its axis along the direction of the Earth's magnetic field of 50.0 μT, and then in 0.200 s it is flipped 180°. An average emf of what magnitude is generated in the coil?

4. A rectangular loop of area A is placed in a region where the magnetic field is perpendicular to the plane of the loop. The magnitude of the field is allowed to vary in time according to $B = B_{\text{max}} e^{-t/\tau}$, where B_{max} and τ are constants. The field has the constant value B_{max} for $t < 0$. (a) Use Faraday's law to show that the emf induced in the loop is given by

$$\mathcal{E} = \frac{AB_{\text{max}}}{\tau} e^{-t/\tau}$$

(b) Obtain a numerical value for $\mathcal{E}$ at $t = 4.00$ s when $A = 0.160$ m^2, $B_{\text{max}} = 0.350$ T, and $\tau = 2.00$ s. (c) For the values of A, B_{max}, and τ given in (b), what is the maximum value of $\mathcal{E}$?

5. A strong electromagnet produces a uniform magnetic field of 1.60 T over a cross-sectional area of 0.200 m^2. We place a coil having 200 turns and a total resistance of 20.0 Ω around the electromagnet. We then smoothly reduce the current in the electromagnet until it reaches zero in 20.0 ms. What is the current induced in the coil?

6. A magnetic field of 0.200 T exists within a solenoid of 500 turns and a diameter of 10.0 cm. How rapidly (that is, within what period of time) must the field be reduced to zero, if the average induced emf within the coil during this time interval is to be 10.0 kV?

7. An aluminum ring of radius 5.00 cm and resistance 3.00×10^{-4} Ω is placed on top of a long air-core solenoid with 1 000 turns per meter and radius 3.00 cm, as shown in Figure P31.7. Over the area of the end of the solenoid, assume that the axial component of the field produced by the solenoid is half as strong as at the center of the solenoid. Assume the solenoid produces negligible field outside its cross-sectional area. The current in the solenoid is increasing at a rate of 270 A/s. (a) What is the induced current in the ring? At the center of the ring, what are (b) the magnitude and (c) the direction of the magnetic field produced by the induced current in the ring?

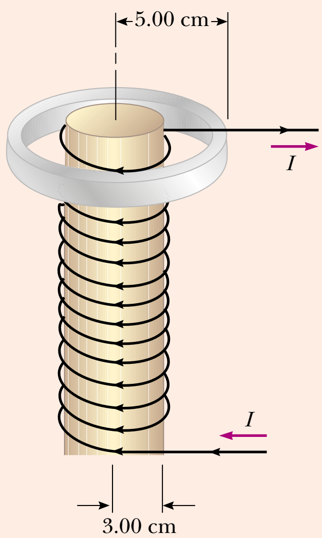

Figure P31.7 Problems 7 and 8.

8. An aluminum ring of radius r_1 and resistance R is placed around the top of a long air-core solenoid with n turns per meter and smaller radius r_2 as shown in Figure P31.7. Assume that the axial component of the field produced by the solenoid over the area of the end of the solenoid is half as strong as at the center of the solenoid. Assume that the solenoid produces negligible field outside its cross-sectional area. The current in the solenoid is increasing at a rate of $\Delta I/\Delta t$. (a) What is the induced current in the ring? (b) At the center of the ring, what is the magnetic field produced by the induced current in the ring? (c) What is the direction of this field?

9. (a) A loop of wire in the shape of a rectangle of width w and length L and a long, straight wire carrying a current I lie on a tabletop as shown in Figure P31.9. (a) Determine the magnetic flux through the loop due to the current I. (b) Suppose the current is changing with time according to $I = a + bt$, where a and b are constants. Determine the emf that is induced in the loop if $b = 10.0$ A/s, $h = 1.00$ cm, $w = 10.0$ cm, and $L = 100$ cm. What is the direction of the induced current in the rectangle?

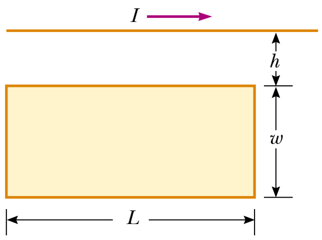

Figure P31.9 Problems 9 and 71.

10. A coil of 15 turns and radius 10.0 cm surrounds a long solenoid of radius 2.00 cm and 1.00×10^3 turns/meter (Fig. P31.10). The current in the solenoid changes as $I = (5.00 \text{ A}) \sin(120t)$. Find the induced emf in the 15-turn coil as a function of time.

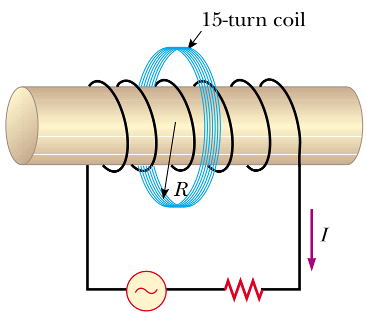

Figure P31.10

11. Find the current through section PQ of length $a = 65.0$ cm in Figure P31.11. The circuit is located in a magnetic field whose magnitude varies with time according to the expression $B = (1.00 \times 10^{-3} \text{ T/s})t$. Assume the resistance per length of the wire is $0.100\ \Omega/\text{m}$.

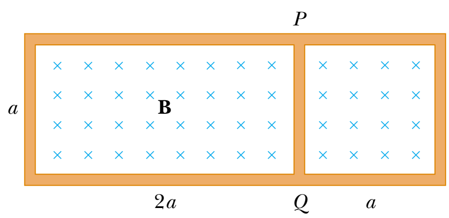

Figure P31.11

12. A 30-turn circular coil of radius 4.00 cm and resistance $1.00\ \Omega$ is placed in a magnetic field directed perpendicular to the plane of the coil. The magnitude of the magnetic field varies in time according to the expression $B = 0.010\,0t + 0.040\,0t^2$, where t is in seconds and B is in tesla. Calculate the induced emf in the coil at $t = 5.00$ s.

13. A long solenoid has $n = 400$ turns per meter and carries a current given by $I = (30.0 \text{ A})(1 - e^{-1.60t})$. Inside the solenoid and coaxial with it is a coil that has a radius of 6.00 cm and consists of a total of $N = 250$ turns of fine wire (Fig. P31.13). What emf is induced in the coil by the changing current?

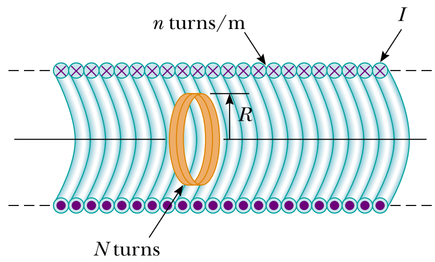

Figure P31.13

14. An instrument based on induced emf has been used to measure projectile speeds up to 6 km/s. A small magnet is imbedded in the projectile, as shown in Figure P31.14. The projectile passes through two coils separated by a distance d. As the projectile passes through each coil a pulse of emf is induced in the coil. The time interval between pulses can be measured accurately with an oscilloscope, and thus the speed can be determined. (a) Sketch a graph of ΔV versus t for the arrangement shown. Consider a current that flows counterclockwise as viewed from the starting point of the projectile as positive. On your graph, indicate which pulse is from coil 1 and which is from coil 2. (b) If the pulse separation is 2.40 ms and $d = 1.50$ m, what is the projectile speed?

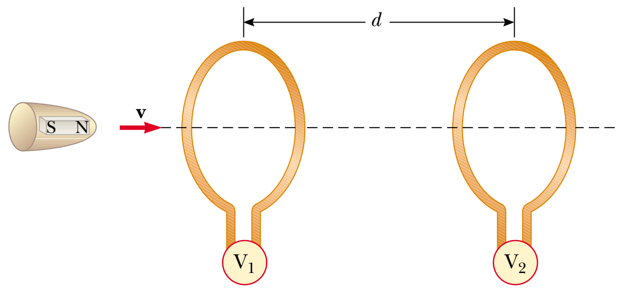

Figure P31.14

15. A coil formed by wrapping 50 turns of wire in the shape of a square is positioned in a magnetic field so that the normal to the plane of the coil makes an angle of 30.0° with the direction of the field. When the magnetic field is increased uniformly from 200 μT to 600 μT in 0.400 s, an emf of magnitude 80.0 mV is induced in the coil. What is the total length of the wire?

16. When a wire carries an AC current with a known frequency, you can use a *Rogowski coil* to determine the amplitude I_{max} of the current without disconnecting the wire to shunt the

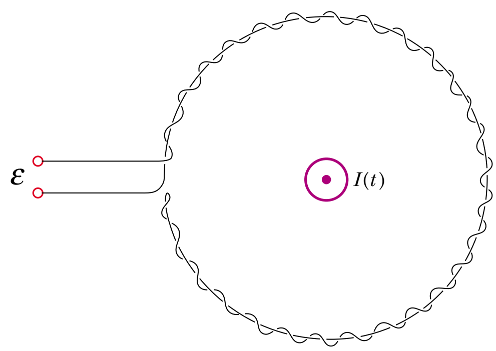

Figure P31.16

current in a meter. The Rogowski coil, shown in Figure P31.16, simply clips around the wire. It consists of a toroidal conductor wrapped around a circular return cord. The toroid has n turns per unit length and a cross-sectional area A. The current to be measured is given by $I(t) = I_{max} \sin \omega t$. (a) Show that the amplitude of the emf induced in the Rogowski coil is $\mathcal{E}_{max} = \mu_0 n A \omega I_{max}$. (b) Explain why the wire carrying the unknown current need not be at the center of the Rogowski coil, and why the coil will not respond to nearby currents that it does not enclose.

17. A toroid having a rectangular cross section ($a = 2.00$ cm by $b = 3.00$ cm) and inner radius $R = 4.00$ cm consists of 500 turns of wire that carries a sinusoidal current $I = I_{max} \sin \omega t$, with $I_{max} = 50.0$ A and a frequency $f = \omega/2\pi = 60.0$ Hz. A coil that consists of 20 turns of wire links with the toroid, as in Figure P31.17. Determine the emf induced in the coil as a function of time.

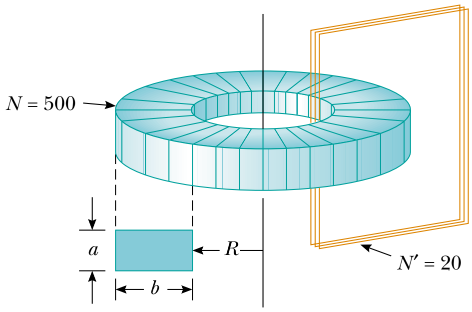

Figure P31.17

18. A piece of insulated wire is shaped into a figure 8, as in Figure P31.18. The radius of the upper circle is 5.00 cm and that of the lower circle is 9.00 cm. The wire has a uniform resistance per unit length of 3.00 Ω/m. A uniform magnetic field is applied perpendicular to the plane of the two circles, in the direction shown. The magnetic field is increasing at a constant rate of 2.00 T/s. Find the magnitude and direction of the induced current in the wire.

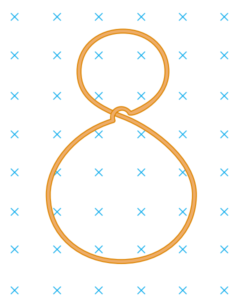
Figure P31.18

Section 31.2 Motional emf
Section 31.3 Lenz's Law

Problem 71 in Chapter 29 can be assigned with this section.

19. An automobile has a vertical radio antenna 1.20 m long. The automobile travels at 65.0 km/h on a horizontal road where the Earth's magnetic field is 50.0 μT directed toward the north and downward at an angle of 65.0° below the horizontal. (a) Specify the direction that the automobile should move in order to generate the maximum motional emf in the antenna, with the top of the antenna positive relative to the bottom. (b) Calculate the magnitude of this induced emf.

20. Consider the arrangement shown in Figure P31.20. Assume that $R = 6.00\ \Omega$, $\ell = 1.20$ m, and a uniform 2.50-T magnetic field is directed into the page. At what speed should the bar be moved to produce a current of 0.500 A in the resistor?

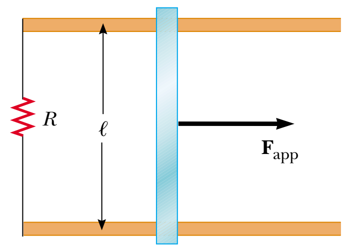

Figure P31.20 Problems 20, 21, and 22.

21. Figure P31.20 shows a top view of a bar that can slide without friction. The resistor is 6.00 Ω and a 2.50-T magnetic field is directed perpendicularly downward, into the paper. Let $\ell = 1.20$ m. (a) Calculate the applied force required to move the bar to the right at a constant speed of 2.00 m/s. (b) At what rate is energy delivered to the resistor?

22. A conducting rod of length ℓ moves on two horizontal, frictionless rails, as shown in Figure P31.20. If a constant force of 1.00 N moves the bar at 2.00 m/s through a magnetic field $\mathbf{B}$ that is directed into the page, (a) what is the current through the 8.00-Ω resistor R? (b) What is the rate at which energy is delivered to the resistor? (c) What is the mechanical power delivered by the force $\mathbf{F}_{app}$?

23. Very large magnetic fields can be produced using a procedure called *flux compression.* A metallic cylindrical tube of radius R is placed coaxially in a long solenoid of somewhat larger radius. The space between the tube and the solenoid is filled with a highly explosive material. When the explosive is set off, it collapses the tube to a cylinder of radius $r < R$. If the collapse happens very rapidly, induced current in the tube maintains the magnetic flux nearly constant inside the tube. If the initial magnetic field in the solenoid is 2.50 T, and $R/r = 12.0$, what maximum value of magnetic field can be achieved?

24. The *homopolar generator,* also called the *Faraday disk,* is a low-voltage, high-current electric generator. It consists of a rotating conducting disk with one stationary brush (a sliding electrical contact) at its axle and another at a point on its circumference, as shown in Figure P31.24. A magnetic field is applied perpendicular to the plane of the disk. Assume the field is 0.900 T, the angular speed is 3 200 rev/min, and the radius of the disk is 0.400 m. Find the emf generated between the brushes. When superconducting coils are used to produce a large magnetic field, a homopolar generator can have a power output of several megawatts. Such a generator is useful, for example, in purifying metals by electrolysis. If a voltage is applied to the output terminals of the generator, it runs in reverse as a *homopolar motor* capable of providing great torque, useful in ship propulsion.

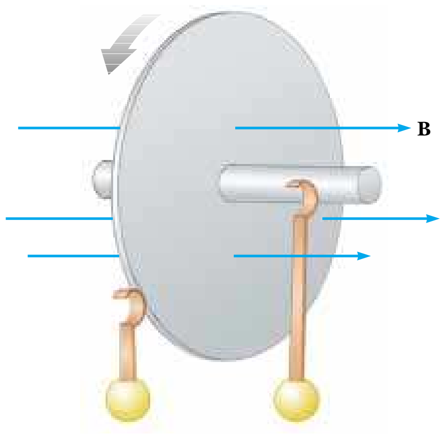

Figure P31.24

25. **Review problem.** A flexible metallic wire with linear density 3.00×10^{-3} kg/m is stretched between two fixed clamps 64.0 cm apart and held under tension 267 N. A magnet is placed near the wire as shown in Figure P31.25. Assume that the magnet produces a uniform field of 4.50 mT over a 2.00-cm length at the center of the wire, and a negligible field elsewhere. The wire is set vibrating at its fundamental (lowest) frequency. The section of the wire in the magnetic field moves with a uniform amplitude of 1.50 cm. Find (a) the frequency and (b) the amplitude of the electromotive force induced between the ends of the wire.

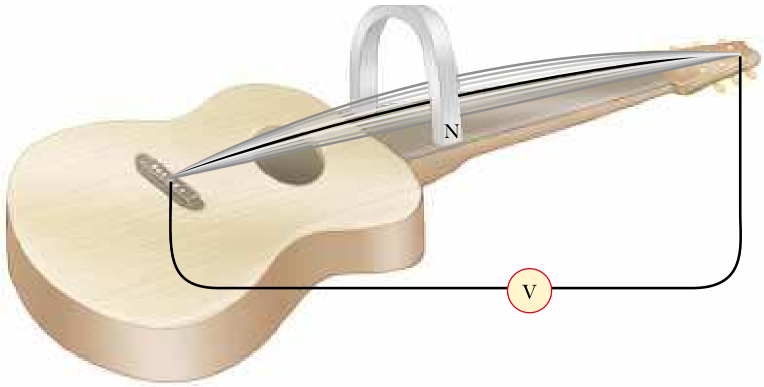

Figure P31.25

26. The square loop in Figure P31.26 is made of wires with total series resistance 10.0 Ω. It is placed in a uniform 0.100-T magnetic field directed perpendicularly into the plane of the paper. The loop, which is hinged at each corner, is pulled as shown until the separation between points A and B is 3.00 m. If this process takes 0.100 s, what is the average current generated in the loop? What is the direction of the current?

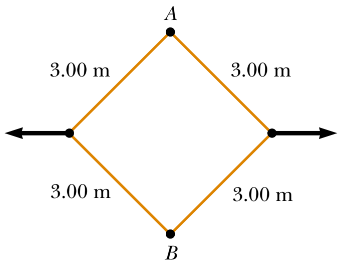

Figure P31.26

27. A helicopter (Figure P31.27) has blades of length 3.00 m, extending out from a central hub and rotating at 2.00 rev/s. If the vertical component of the Earth's magnetic field is 50.0 μT, what is the emf induced between the blade tip and the center hub?

Ross Harrison Koty/Getty Images

Figure P31.27

28. Use Lenz's law to answer the following questions concerning the direction of induced currents. (a) What is the direction of the induced current in resistor R in Figure P31.28a when the bar magnet is moved to the left? (b) What is the direction of the current induced in the resistor R immediately after the switch S in Figure P31.28b

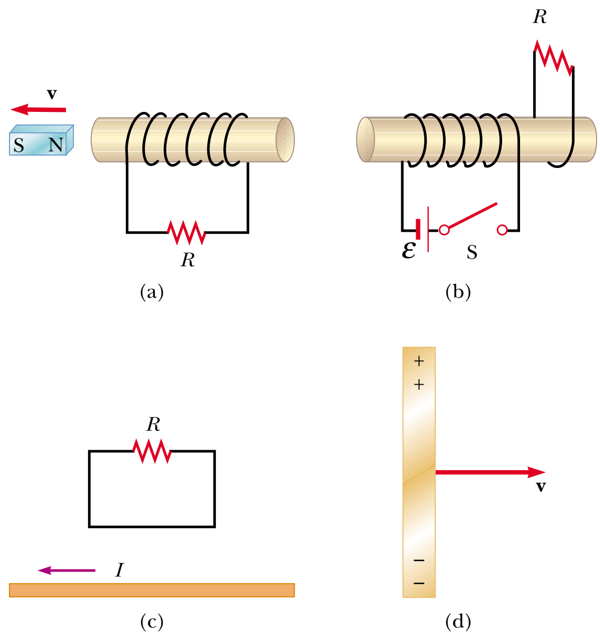

Figure P31.28

is closed? (c) What is the direction of the induced current in R when the current I in Figure P31.28c decreases rapidly to zero? (d) A copper bar is moved to the right while its axis is maintained in a direction perpendicular to a magnetic field, as shown in Figure P31.28d. If the top of the bar becomes positive relative to the bottom, what is the direction of the magnetic field?

29. A rectangular coil with resistance R has N turns, each of length ℓ and width w as shown in Figure P31.29. The coil moves into a uniform magnetic field $\mathbf{B}$ with constant velocity $\mathbf{v}$. What are the magnitude and direction of the total magnetic force on the coil (a) as it enters the magnetic field, (b) as it moves within the field, and (c) as it leaves the field?

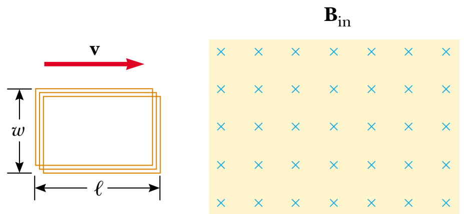

Figure P31.29

30. In Figure P31.30, the bar magnet is moved toward the loop. Is $V_a - V_b$ positive, negative, or zero? Explain.

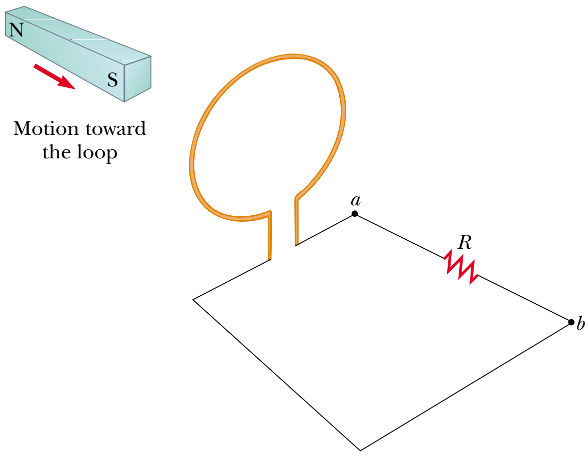

Figure P31.30

31. Two parallel rails with negligible resistance are 10.0 cm apart and are connected by a 5.00-Ω resistor. The circuit also contains two metal rods having resistances of 10.0 Ω and 15.0 Ω sliding along the rails (Fig. P31.31). The rods are pulled away from the resistor at constant speeds of 4.00 m/s and 2.00 m/s, respectively. A uniform magnetic field of magnitude 0.010 0 T is applied perpendicular to the plane of the rails. Determine the current in the 5.00-Ω resistor.

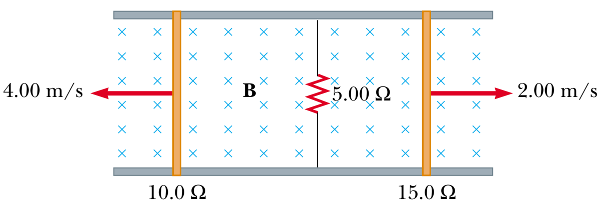

Figure P31.31

Section 31.4 Induced emf and Electric Fields

32. For the situation shown in Figure P31.32, the magnetic field changes with time according to the expression $B = (2.00t^3 - 4.00t^2 + 0.800)\text{T}$, and $r_2 = 2R = 5.00$ cm. (a) Calculate the magnitude and direction of the force exerted on an electron located at point P_2 when $t = 2.00$ s. (b) At what time is this force equal to zero?

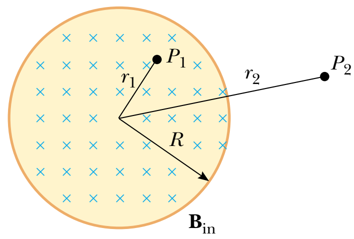

Figure P31.32 Problems 32 and 33.

33. A magnetic field directed into the page changes with time according to $B = (0.030\,0t^2 + 1.40)\text{T}$, where t is in seconds. The field has a circular cross section of radius $R = 2.50$ cm (Fig. P31.32). What are the magnitude and direction of the electric field at point P_1 when $t = 3.00$ s and $r_1 = 0.020\,0$ m?

34. A long solenoid with 1 000 turns per meter and radius 2.00 cm carries an oscillating current given by $I = (5.00 \text{ A}) \sin(100\pi t)$. What is the electric field induced at a radius $r = 1.00$ cm from the axis of the solenoid? What is the direction of this electric field when the current is increasing counterclockwise in the coil?

Section 31.5 Generators and Motors

Problems 28 and 62 in Chapter 29 can be assigned with this section.

35. A coil of area 0.100 m^2 is rotating at 60.0 rev/s with the axis of rotation perpendicular to a 0.200-T magnetic field. (a) If the coil has 1 000 turns, what is the maximum emf generated in it? (b) What is the orientation of the coil with respect to the magnetic field when the maximum induced voltage occurs?

36. In a 250-turn automobile alternator, the magnetic flux in each turn is $\Phi_B = (2.50 \times 10^{-4} \text{ Wb}) \cos(\omega t)$, where ω is the angular speed of the alternator. The alternator is geared to rotate three times for each engine revolution. When the engine is running at an angular speed of 1 000 rev/min, determine (a) the induced emf in the alternator as a function of time and (b) the maximum emf in the alternator.

37. A long solenoid, with its axis along the x axis, consists of 200 turns per meter of wire that carries a steady current of 15.0 A. A coil is formed by wrapping 30 turns of thin wire around a circular frame that has a radius of 8.00 cm. The coil is placed inside the solenoid and mounted on an axis that is a diameter of the coil and coincides with the y axis. The coil is then rotated with an angular speed of 4.00π rad/s. (The plane of the coil is in the yz plane at $t = 0$.) Determine the emf generated in the coil as a function of time.

38. A bar magnet is spun at constant angular speed ω around an axis as shown in Figure P31.38. A stationary flat rectangular conducting loop surrounds the magnet, and at $t = 0$, the magnet is oriented as shown. Make a qualitative graph of the induced current in the loop as a function of time, plotting counterclockwise currents as positive and clockwise currents as negative.

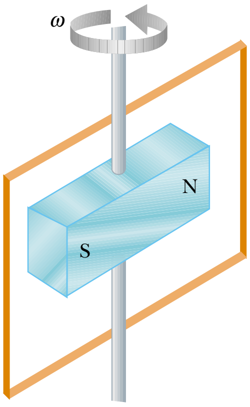

Figure P31.38

39. A motor in normal operation carries a direct current of 0.850 A when connected to a 120-V power supply. The resistance of the motor windings is 11.8 Ω. While in normal operation, (a) what is the back emf generated by the motor? (b) At what rate is internal energy produced in the windings? (c) **What If?** Suppose that a malfunction stops the motor shaft from rotating. At what rate will internal energy be produced in the windings in this case? (Most motors have a thermal switch that will turn off the motor to prevent overheating when this occurs.)

40. A semicircular conductor of radius $R = 0.250$ m is rotated about the axis AC at a constant rate of 120 rev/min (Fig. P31.40). A uniform magnetic field in all of the lower half of the figure is directed out of the plane of rotation and has a magnitude of 1.30 T. (a) Calculate the maximum value of the emf induced in the conductor. (b) What is the value of the average induced emf for each complete rotation? (c) **What If?** How would the answers to (a) and (b) change if **B** were allowed to extend a distance R above the axis of rotation? Sketch the emf versus time (d) when the field is as drawn in Figure P31.40 and (e) when the field is extended as described in (c).

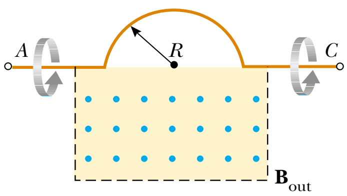

Figure P31.40

41. The rotating loop in an AC generator is a square 10.0 cm on a side. It is rotated at 60.0 Hz in a uniform field of 0.800 T. Calculate (a) the flux through the loop as a function of time, (b) the emf induced in the loop, (c) the current induced in the loop for a loop resistance of 1.00 Ω, (d) the power delivered to the loop, and (e) the torque that must be exerted to rotate the loop.

Section 31.6 Eddy Currents

42. Figure P31.42 represents an electromagnetic brake that uses eddy currents. An electromagnet hangs from a railroad car near one rail. To stop the car, a large current is sent through the coils of the electromagnet. The moving electromagnet induces eddy currents in the rails, whose fields oppose the change in the field of the electromagnet. The magnetic fields of the eddy currents exert force on the current in the electromagnet, thereby slowing the car. The direction of the car's motion and the direction of the current in the electromagnet are shown correctly in the picture. Determine which of the eddy currents shown on the rails is correct. Explain your answer.

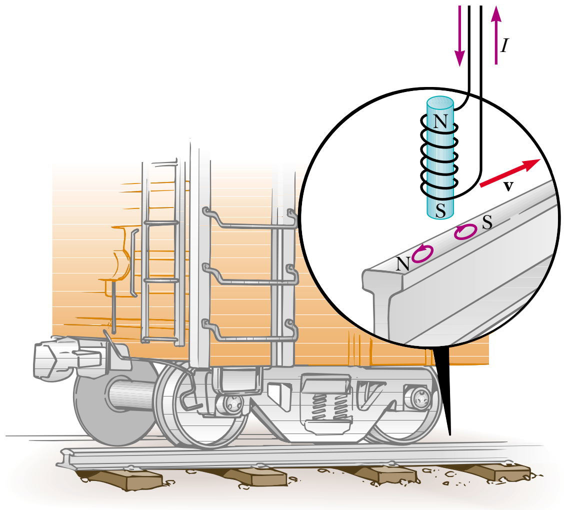

Figure P31.42

43. A conducting rectangular loop of mass M, resistance R, and dimensions w by ℓ falls from rest into a magnetic field **B** as shown in Figure P31.43. During the time interval

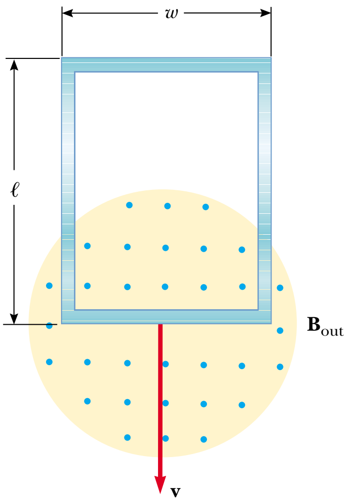

Figure P31.43

before the top edge of the loop reaches the field, the loop approaches a terminal speed v_T. (a) Show that

$$v_T = \frac{MgR}{B^2w^2}$$

(b) Why is v_T proportional to R? (c) Why is it inversely proportional to B^2?

Section 31.7 Maxwell's Equations

44. An electron moves through a uniform electric field $\mathbf{E} = (2.50\hat{\mathbf{i}} + 5.00\hat{\mathbf{j}})$ V/m and a uniform magnetic field $\mathbf{B} = (0.400\hat{\mathbf{k}})$T. Determine the acceleration of the electron when it has a velocity $\mathbf{v} = 10.0\hat{\mathbf{i}}$ m/s.

45. A proton moves through a uniform electric field given by $\mathbf{E} = 50.0\hat{\mathbf{j}}$ V/m and a uniform magnetic field $\mathbf{B} = (0.200\hat{\mathbf{i}} + 0.300\hat{\mathbf{j}} + 0.400\hat{\mathbf{k}})$T. Determine the acceleration of the proton when it has a velocity $\mathbf{v} = 200\hat{\mathbf{i}}$ m/s.

Additional Problems

46. A steel guitar string vibrates (Figure 31.6). The component of magnetic field perpendicular to the area of a pickup coil nearby is given by

$$B = 50.0\text{ mT} + (3.20\text{ mT})\sin(2\pi 523\, t/\text{s})$$

The circular pickup coil has 30 turns and radius 2.70 mm. Find the emf induced in the coil as a function of time.

47. Figure P31.47 is a graph of the induced emf versus time for a coil of N turns rotating with angular speed ω in a uniform magnetic field directed perpendicular to the axis of rotation of the coil. **What If?** Copy this sketch (on a larger scale), and on the same set of axes show the graph of emf versus t (a) if the number of turns in the coil is doubled; (b) if instead the angular speed is doubled; and (c) if the angular speed is doubled while the number of turns in the coil is halved.

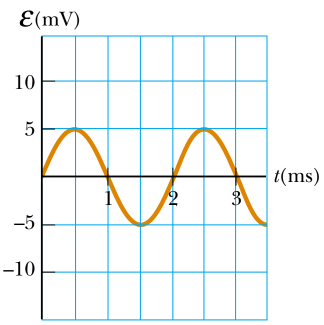

Figure P31.47

48. A technician wearing a brass bracelet enclosing area 0.005 00 m^2 places her hand in a solenoid whose magnetic field is 5.00 T directed perpendicular to the plane of the bracelet. The electrical resistance around the circumference of the bracelet is 0.020 0 Ω. An unexpected power failure causes the field to drop to 1.50 T in a time of 20.0 ms. Find (a) the current induced in the bracelet and (b) the power delivered to the bracelet. *Note:* As this problem implies, you should not wear any metal objects when working in regions of strong magnetic fields.

49. Two infinitely long solenoids (seen in cross section) pass through a circuit as shown in Figure P31.49. The magnitude of $\mathbf{B}$ inside each is the same and is increasing at the rate of 100 T/s. What is the current in each resistor?

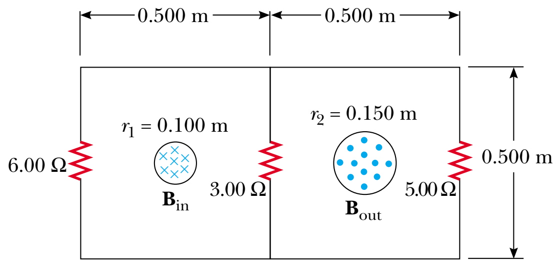

Figure P31.49

50. A conducting rod of length $\ell = 35.0$ cm is free to slide on two parallel conducting bars as shown in Figure P31.50. Two resistors $R_1 = 2.00$ Ω and $R_2 = 5.00$ Ω are connected across the ends of the bars to form a loop. A constant magnetic field $B = 2.50$ T is directed perpendicularly into the page. An external agent pulls the rod to the left with a constant speed of $v = 8.00$ m/s. Find (a) the currents in both resistors, (b) the total power delivered to the resistance of the circuit, and (c) the magnitude of the applied force that is needed to move the rod with this constant velocity.

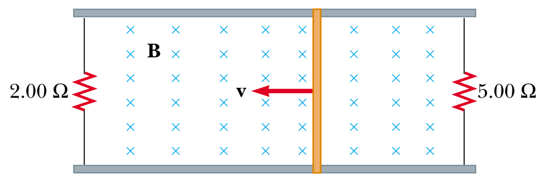

Figure P31.50

51. Suppose you wrap wire onto the core from a roll of cellophane tape to make a coil. Describe how you can use a bar magnet to produce an induced voltage in the coil. What is the order of magnitude of the emf you generate? State the quantities you take as data and their values.

52. A bar of mass m, length d, and resistance R slides without friction in a horizontal plane, moving on parallel rails as shown in Figure P31.52. A battery that maintains a constant emf $\mathcal{E}$ is connected between the rails, and a constant magnetic field $\mathbf{B}$ is directed perpendicularly to the plane of the page. Assuming the bar starts from rest, show that at time t it moves with a speed

$$v = \frac{\mathcal{E}}{Bd}\left(1 - e^{-B^2d^2t/mR}\right)$$

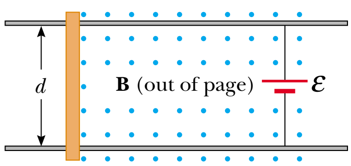

Figure P31.52

53. **Review problem**. A particle with a mass of 2.00×10^{-16} kg and a charge of 30.0 nC starts from rest, is accelerated by a strong electric field, and is fired from a small source inside a region of uniform constant magnetic field 0.600 T. The velocity of the particle is perpendicular to the field. The circular orbit of the particle encloses a magnetic flux of 15.0 μWb. (a) Calculate the speed of the particle. (b) Calculate the potential difference through which the particle accelerated inside the source.

54. An *induction furnace* uses electromagnetic induction to produce eddy currents in a conductor, thereby raising the conductor's temperature. Commercial units operate at frequencies ranging from 60 Hz to about 1 MHz and deliver powers from a few watts to several megawatts. Induction heating can be used for welding in a vacuum enclosure, to avoid oxidation and contamination of the metal. At high frequencies, induced currents occur only near the surface of the conductor—this is the "skin effect." By creating an induced current for a short time at an appropriately high frequency, one can heat a sample down to a controlled depth. For example, the surface of a farm tiller can be tempered to make it hard and brittle for effective cutting while keeping the interior metal soft and ductile to resist breakage.

To explore induction heating, consider a flat conducting disk of radius R, thickness b, and resistivity ρ. A sinusoidal magnetic field $B_{max} \cos \omega t$ is applied perpendicular to the disk. Assume that the frequency is so low that the skin effect is not important. Assume the eddy currents occur in circles concentric with the disk. (a) Calculate the average power delivered to the disk. (b) **What If?** By what factor does the power change when the amplitude of the field doubles? (c) When the frequency doubles? (d) When the radius of the disk doubles?

55. The plane of a square loop of wire with edge length $a = 0.200$ m is perpendicular to the Earth's magnetic field at a point where $B = 15.0\ \mu$T, as shown in Figure P31.55. The total resistance of the loop and the wires connecting it to a sensitive ammeter is 0.500 Ω. If the loop is suddenly collapsed by horizontal forces as shown, what total charge passes through the ammeter?

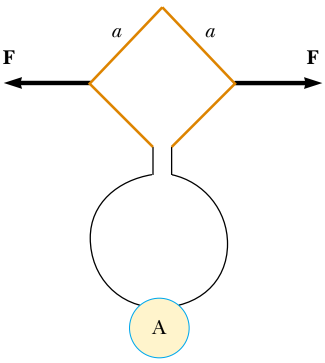

Figure P31.55

56. Magnetic field values are often determined by using a device known as a *search coil.* This technique depends on the measurement of the total charge passing through a coil in a time interval during which the magnetic flux linking the windings changes either because of the motion of the coil or because of a change in the value of B. (a) Show that as the flux through the coil changes from Φ_1 to Φ_2, the charge transferred through the coil will be given by $Q = N(\Phi_2 - \Phi_1)/R$, where R is the resistance of the coil and a sensitive ammeter connected across it and N is the number of turns. (b) As a specific example, calculate B when a 100-turn coil of resistance 200 Ω and cross-sectional area 40.0 cm^2 produces the following results. A total charge of 5.00×10^{-4} C passes through the coil when it is rotated in a uniform field from a position where the plane of the coil is perpendicular to the field to a position where the coil's plane is parallel to the field.

57. In Figure P31.57, the rolling axle, 1.50 m long, is pushed along horizontal rails at a constant speed $v = 3.00$ m/s. A resistor $R = 0.400\ \Omega$ is connected to the rails at points a and b, directly opposite each other. (The wheels make good electrical contact with the rails, and so the axle, rails, and R form a closed-loop circuit. The only significant resistance in the circuit is R.) A uniform magnetic field $B = 0.080\,0$ T is vertically downward. (a) Find the induced current I in the resistor. (b) What horizontal force F is required to keep the axle rolling at constant speed? (c) Which end of the resistor, a or b, is at the higher electric potential? (d) **What If?** After the axle rolls past the resistor, does the current in R reverse direction? Explain your answer.

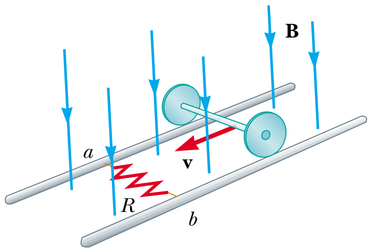

Figure P31.57

58. A conducting rod moves with a constant velocity **v** in a direction perpendicular to a long, straight wire carrying a current I as shown in Figure P31.58. Show that the magnitude of the emf generated between the ends of the rod is

$$|\mathcal{E}| = \frac{\mu_0 v I \ell}{2\pi r}$$

In this case, note that the emf decreases with increasing r, as you might expect.

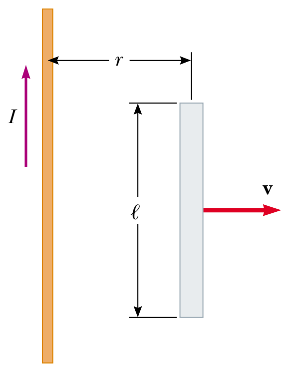

Figure P31.58

59. A circular loop of wire of radius r is in a uniform magnetic field, with the plane of the loop perpendicular to the direction of the field (Fig. P31.59). The magnetic field varies with time according to $B(t) = a + bt$, where a and b are constants. (a) Calculate the magnetic flux through the loop at $t = 0$. (b) Calculate the emf induced in the loop. (c) If the resistance of the loop is R, what is the induced current? (d) At what rate is energy being delivered to the resistance of the loop?

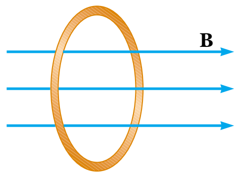

Figure P31.59

60. In Figure P31.60, a uniform magnetic field decreases at a constant rate $dB/dt = -K$, where K is a positive constant. A circular loop of wire of radius a containing a resistance R and a capacitance C is placed with its plane normal to the field. (a) Find the charge Q on the capacitor when it is fully charged. (b) Which plate is at the higher potential? (c) Discuss the force that causes the separation of charges.

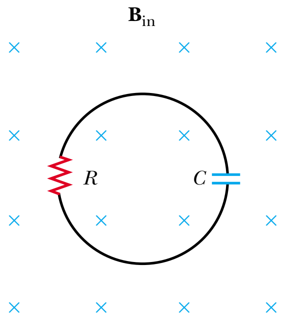

Figure P31.60

61. A rectangular coil of 60 turns, dimensions 0.100 m by 0.200 m and total resistance 10.0 Ω, rotates with angular speed 30.0 rad/s about the y axis in a region where a 1.00-T magnetic field is directed along the x axis. The rotation is initiated so that the plane of the coil is perpendicular to the direction of $\mathbf{B}$ at $t = 0$. Calculate (a) the maximum induced emf in the coil, (b) the maximum rate of change of magnetic flux through the coil, (c) the induced emf at $t = 0.050\,0$ s, and (d) the torque exerted by the magnetic field on the coil at the instant when the emf is a maximum.

62. A small circular washer of radius 0.500 cm is held directly below a long, straight wire carrying a current of 10.0 A. The washer is located 0.500 m above the top of a table (Fig. P31.62). (a) If the washer is dropped from rest, what is the magnitude of the average induced emf in the washer from the time it is released to the moment it hits the tabletop? Assume that the magnetic field is nearly constant over the area of the washer, and equal to the magnetic field at the center of the washer. (b) What is the direction of the induced current in the washer?

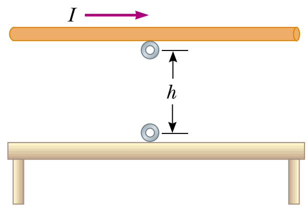

Figure P31.62

63. A conducting rod of length ℓ moves with velocity $\mathbf{v}$ parallel to a long wire carrying a steady current I. The axis of the rod is maintained perpendicular to the wire with the near end a distance r away, as shown in Figure P31.63. Show that the magnitude of the emf induced in the rod is

$$|\mathcal{E}| = \frac{\mu_0 I v}{2\pi} \ln\left(1 + \frac{\ell}{r}\right)$$

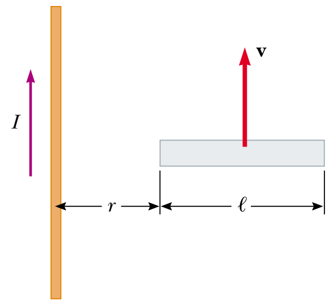

Figure P31.63

64. A rectangular loop of dimensions ℓ and w moves with a constant velocity $\mathbf{v}$ away from a long wire that carries a current I in the plane of the loop (Fig. P31.64). The total resistance of the loop is R. Derive an expression that gives the current in the loop at the instant the near side is a distance r from the wire.

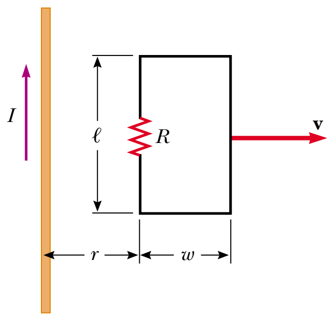

Figure P31.64

65. The magnetic flux through a metal ring varies with time t according to $\Phi_B = 3(at^3 - bt^2)\ \text{T}\cdot\text{m}^2$, with $a = 2.00\ \text{s}^{-3}$ and $b = 6.00\ \text{s}^{-2}$. The resistance of the ring is 3.00 Ω. Determine the maximum current induced in the ring during the interval from $t = 0$ to $t = 2.00$ s.

66. **Review problem.** The bar of mass m in Figure P31.66 is pulled horizontally across parallel rails by a massless string that passes over an ideal pulley and is attached to a suspended object of mass M. The uniform magnetic field has a magnitude B, and the distance between the rails is ℓ. The rails are connected at one end by a load resistor R. Derive an expression that gives the horizontal speed of the bar as a function of time, assuming that the suspended object is released with the bar at rest at $t = 0$. Assume no friction between rails and bar.

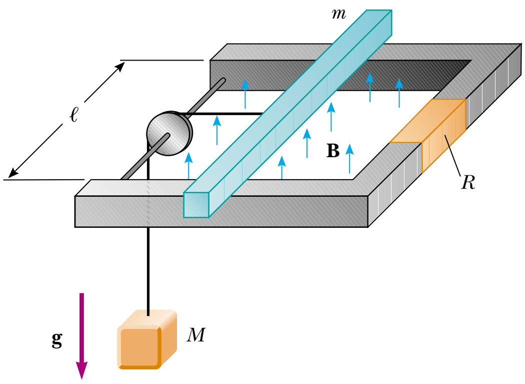

Figure P31.66

67. A solenoid wound with 2 000 turns/m is supplied with current that varies in time according to $I = (4\text{A})\sin(120\pi t)$, where t is in seconds. A small coaxial circular coil of 40 turns and radius $r = 5.00$ cm is located inside the solenoid near its center. (a) Derive an expression that describes the manner in which the emf in the small coil varies in time. (b) At what average rate is energy delivered to the small coil if the windings have a total resistance of 8.00 Ω?

68. Figure P31.68 shows a stationary conductor whose shape is similar to the letter e. The radius of its circular portion is $a = 50.0$ cm. It is placed in a constant magnetic field of 0.500 T directed out of the page. A straight conducting rod, 50.0 cm long, is pivoted about point O and rotates with a constant angular speed of 2.00 rad/s. (a) Determine the induced emf in the loop POQ. Note that the area of the loop is $\theta a^2/2$. (b) If all of the conducting material has a resistance per length of 5.00 Ω/m, what is the induced current in the loop POQ at the instant 0.250 s after point P passes point Q?

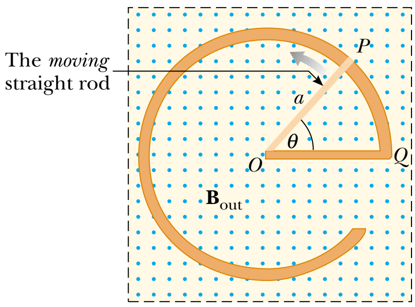

Figure P31.68

69. A *betatron* accelerates electrons to energies in the MeV range by means of electromagnetic induction. Electrons in a vacuum chamber are held in a circular orbit by a magnetic field perpendicular to the orbital plane. The magnetic field is gradually increased to induce an electric field around the orbit. (a) Show that the electric field is in the correct direction to make the electrons speed up. (b) Assume that the radius of the orbit remains constant. Show that the average magnetic field over the area enclosed by the orbit must be twice as large as the magnetic field at the circumference of the circle.

70. A wire 30.0 cm long is held parallel to and 80.0 cm above a long wire carrying 200 A and resting on the floor (Fig. P31.70). The 30.0-cm wire is released and falls, remaining parallel with the current-carrying wire as it falls. Assume that the falling wire accelerates at 9.80 m/s^2 and derive an equation for the emf induced in it. Express your result as a function of the time t after the wire is dropped. What is the induced emf 0.300 s after the wire is released?

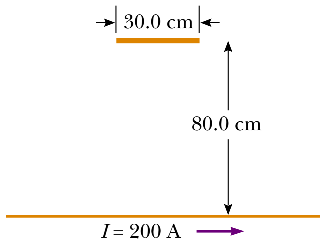

Figure P31.70

71. A long, straight wire carries a current that is given by $I = I_{max}\sin(\omega t + \phi)$ and lies in the plane of a rectangular coil of N turns of wire, as shown in Figure P31.9. The quantities I_{max}, ω, and ϕ are all constants. Determine the emf induced in the coil by the magnetic field created by the current in the straight wire. Assume $I_{max} = 50.0$ A, $\omega = 200\pi\ \text{s}^{-1}$, $N = 100$, $h = w = 5.00$ cm, and $L = 20.0$ cm.

72. A dime is suspended from a thread and hung between the poles of a strong horseshoe magnet as shown in Figure P31.72. The dime rotates at constant angular speed ω

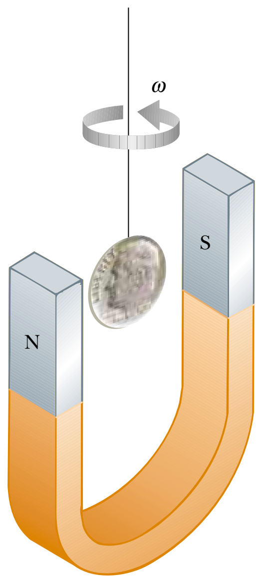

Figure P31.72

about a vertical axis. Letting θ represent the angle between the direction of **B** and the normal to the face of the dime, sketch a graph of the torque due to induced currents as a function of θ for $0 < \theta < 2\pi$.

Answers to Quick Quizzes

31.1 (c). In all cases except this one, there is a change in the magnetic flux through the loop.

31.2 c, $d = e$, b, a. The magnitude of the emf is proportional to the rate of change of the magnetic flux. For the situation described, the rate of change of magnetic flux is proportional to the rate of change of the magnetic field. This rate of change is the slope of the graph in Figure 31.4. The magnitude of the slope is largest at c. Points d and e are on a straight line, so the slope is the same at each point. Point b represents a point of relatively small slope, while a is at a point of zero slope because the curve is horizontal at that point.

31.3 (b). The magnetic field lines around the transmission cable will be circular, centered on the cable. If you place your loop around the cable, there are no field lines passing through the loop, so no emf is induced. The loop must be placed next to the cable, with the plane of the loop parallel to the cable to maximize the flux through its area.

31.4 (a). The Earth's magnetic field has a downward component in the northern hemisphere. As the plane flies north, the right-hand rule illustrated in Figure 29.4 indicates that positive charge experiences a force directed toward the west. Thus, the left wingtip becomes positively charged and the right wingtip negatively charged.

31.5 (c). The force on the wire is of magnitude $F_{\text{app}} = F_B = I\ell B$, with I given by Equation 31.6. Thus, the force is proportional to the speed and the force doubles. Because $\mathcal{P} = F_{\text{app}}v$, the doubling of the force *and* the speed results in the power being four times as large.

31.6 (b). According to Equation 31.5, because B and v are fixed, the emf depends only on the length of the wire moving in the magnetic field. Thus, you want the long dimension moving through the magnetic field lines so that it is perpendicular to the velocity vector. In this case, the short dimension is parallel to the velocity vector.

31.7 (a). Because the current induced in the solenoid is clockwise when viewed from above, the magnetic field lines produced by this current point downward in Figure 31.15. Thus, the upper end of the solenoid acts as a south pole. For this situation to be consistent with Lenz's law, the south pole of the bar magnet must be approaching the solenoid.

31.8 (b). At the position of the loop, the magnetic field lines due to the wire point into the page. The loop is entering a region of stronger magnetic field as it drops toward the wire, so the flux is increasing. The induced current must set up a magnetic field that opposes this increase. To do this, it creates a magnetic field directed out of the page. By the right-hand rule for current loops, this requires a counterclockwise current in the loop.

31.9 (d). The constant rate of change of B will result in a constant rate of change of the magnetic flux. According to Equation 31.9, if $d\Phi_B/dt$ is constant, **E** is constant in magnitude.

31.10 (a). While reducing the resistance may increase the current that the generator provides to a load, it does not alter the emf. Equation 31.11 shows that the emf depends on ω, B, and N, so all other choices increase the emf.

31.11 (b). When the aluminum sheet moves between the poles of the magnet, eddy currents are established in the aluminum. According to Lenz's law, these currents are in a direction so as to oppose the original change, which is the movement of the aluminum sheet in the magnetic field. The same principle is used in common laboratory triple-beam balances. See if you can find the magnet and the aluminum sheet the next time you use a triple-beam balance.