## PROBLEMS

1, 2, 3 = straightforward, intermediate, challenging $\square$ = full solution available in the Student Solutions Manual and Study Guide
2 muw = coached solution with hints available at http://www.pse6.com $\quad \square$ = computer useful in solving problem
$=$ paired numerical and symbolic problems

## Section 27.1 Electric Current

1. In a particular cathode ray tube, the measured beam current is $30.0 \mu \mathrm{~A}$. How many electrons strike the tube screen every 40.0 s ?
2. A teapot with a surface area of $700 \mathrm{~cm}^{2}$ is to be silver plated. It is attached to the negative electrode of an electrolytic cell containing silver nitrate $\left(\mathrm{Ag}^{+} \mathrm{NO}_{3}{ }^{-}\right)$. If the cell is powered by a $12.0-\mathrm{V}$ battery and has a resistance of $1.80 \Omega$, how long does it take for a $0.133-\mathrm{mm}$ layer of silver to build up on the teapot? (The density of silver is $10.5 \times 10^{3} \mathrm{~kg} / \mathrm{m}^{3}$.)
3. 2nve Suppose that the current through a conductor decreases exponentially with time according to the equation $I(t)=I_{0} e^{-t / \tau}$ where $I_{0}$ is the initial current (at $t=0$ ), and $\tau$ is a constant having dimensions of time. Consider a fixed observation point within the conductor. (a) How much charge passes this point between $t=0$ and $t=\tau$ ? (b) How much charge passes this point between $t=0$ and $t=10 \tau$ ? (c) What If? How much charge passes this point between $t=0$ and $t=\infty$ ?
4. In the Bohr model of the hydrogen atom, an electron in the lowest energy state follows a circular path $5.29 \times 10^{-11} \mathrm{~m}$ from the proton. (a) Show that the speed of the electron is $2.19 \times 10^{6} \mathrm{~m} / \mathrm{s}$. (b) What is the effective current associated with this orbiting electron?
5. A small sphere that carries a charge $q$ is whirled in a circle at the end of an insulating string. The angular frequency of rotation is $\omega$. What average current does this rotating charge represent?
6. The quantity of charge $q$ (in coulombs) that has passed through a surface of area $2.00 \mathrm{~cm}^{2}$ varies with time according to the equation $q=4 t^{3}+5 t+6$, where $t$ is in seconds. (a) What is the instantaneous current through the surface at $t=1.00 \mathrm{~s}$ ? (b) What is the value of the current density?
7. An electric current is given by the expression $I(t)=$ $100 \sin (120 \pi t)$, where $I$ is in amperes and $t$ is in seconds. What is the total charge carried by the current from $t=0$ to $t=(1 / 240) \mathrm{s}$ ?
8. Figure P27.8 represents a section of a circular conductor of nonuniform diameter carrying a current of 5.00 A . The


Figure P27.8
radius of cross section $A_{1}$ is 0.400 cm . (a) What is the magnitude of the current density across $A_{1}$ ? (b) If the current density across $A_{2}$ is one-fourth the value across $A_{1}$, what is the radius of the conductor at $A_{2}$ ?
9. The electron beam emerging from a certain high-energy electron accelerator has a circular cross section of radius 1.00 mm . (a) The beam current is $8.00 \mu \mathrm{~A}$. Find the current density in the beam, assuming that it is uniform throughout. (b) The speed of the electrons is so close to the speed of light that their speed can be taken as $c=3.00 \times 10^{8} \mathrm{~m} / \mathrm{s}$ with negligible error. Find the electron density in the beam.
(c) How long does it take for Avogadro's number of electrons to emerge from the accelerator?
10. A Van de Graaff generator produces a beam of $2.00-\mathrm{MeV}$ deuterons, which are heavy hydrogen nuclei containing a proton and a neutron. (a) If the beam current is $10.0 \mu \mathrm{~A}$, how far apart are the deuterons? (b) Is the electric force of repulsion among them a significant factor in beam stability? Explain.
11. An aluminum wire having a cross-sectional area of $4.00 \times 10^{-6} \mathrm{~m}^{2}$ carries a current of 5.00 A . Find the drift speed of the electrons in the wire. The density of aluminum is $2.70 \mathrm{~g} / \mathrm{cm}^{3}$. Assume that one conduction electron is supplied by each atom.

## Section 27.2 Resistance

12. Calculate the current density in a gold wire at $20^{\circ} \mathrm{C}$, if an electric field of $0.740 \mathrm{~V} / \mathrm{m}$ exists in the wire.
13. A lightbulb has a resistance of $240 \Omega$ when operating with a potential difference of 120 V across it. What is the current in the lightbulb?
14. A resistor is constructed of a carbon rod that has a uniform cross-sectional area of $5.00 \mathrm{~mm}^{2}$. When a potential difference of 15.0 V is applied across the ends of the rod, the rod carries a current of $4.00 \times 10^{-3} \mathrm{~A}$. Find (a) the resistance of the rod and (b) the rod's length.
15. A $0.900-\mathrm{V}$ potential difference is maintained across a $1.50-\mathrm{m}$ length of tungsten wire that has a cross-sectional area of $0.600 \mathrm{~mm}^{2}$. What is the current in the wire?
16. A conductor of uniform radius 1.20 cm carries a current of 3.00 A produced by an electric field of $120 \mathrm{~V} / \mathrm{m}$. What is the resistivity of the material?
17. Suppose that you wish to fabricate a uniform wire out of 1.00 g of copper. If the wire is to have a resistance of $R=0.500 \Omega$, and if all of the copper is to be used, what will be (a) the length and (b) the diameter of this wire?
18. Gold is the most ductile of all metals. For example, one gram of gold can be drawn into a wire 2.40 km long. What is the resistance of such a wire at $20^{\circ} \mathrm{C}$ ? You can find the necessary reference information in this textbook.
19. (a) Make an order-of-magnitude estimate of the resistance between the ends of a rubber band. (b) Make an order-of-magnitude estimate of the resistance between the 'heads' and 'tails' sides of a penny. In each case state what quantities you take as data and the values you measure or estimate for them. (c) WARNING! Do not try this at home! What is the order of magnitude of the current that each would carry if it were connected across a $120-\mathrm{V}$ power supply?
20. A solid cube of silver (density $=10.5 \mathrm{~g} / \mathrm{cm}^{3}$ ) has a mass of 90.0 g . (a) What is the resistance between opposite faces of the cube? (b) Assume each silver atom contributes one conduction electron. Find the average drift speed of electrons when a potential difference of $1.00 \times 10^{-5} \mathrm{~V}$ is applied to opposite faces. The atomic number of silver is 47 , and its molar mass is $107.87 \mathrm{~g} / \mathrm{mol}$.
21. A metal wire of resistance $R$ is cut into three equal pieces that are then connected side by side to form a new wire the length of which is equal to one-third the original length. What is the resistance of this new wire?
22. Aluminum and copper wires of equal length are found to have the same resistance. What is the ratio of their radii?
23. A current density of $6.00 \times 10^{-13} \mathrm{~A} / \mathrm{m}^{2}$ exists in the atmosphere at a location where the electric field is $100 \mathrm{~V} / \mathrm{m}$. Calculate the electrical conductivity of the Earth's atmosphere in this region.
24. The rod in Figure P27.24 is made of two materials. The figure is not drawn to scale. Each conductor has a square cross section 3.00 mm on a side. The first material has a resistivity of $4.00 \times 10^{-3} \Omega \cdot \mathrm{~m}$ and is 25.0 cm long, while the second material has a resistivity of $6.00 \times 10^{-3} \Omega \cdot \mathrm{~m}$ and is 40.0 cm long. What is the resistance between the ends of the rod?


Figure P27.24

## Section 27.3 A Model for Electrical Conduction

25.200 If the magnitude of the drift velocity of free electrons in a copper wire is $7.84 \times 10^{-4} \mathrm{~m} / \mathrm{s}$, what is the electric field in the conductor?
26. If the current carried by a conductor is doubled, what happens to the (a) charge carrier density? (b) current density? (c) electron drift velocity? (d) average time interval between collisions?
27. Use data from Example 27.1 to calculate the collision mean free path of electrons in copper. Assume the average thermal speed of conduction electrons is $8.60 \times 10^{5} \mathrm{~m} / \mathrm{s}$.

## Section 27.4 Resistance and Temperature

28. While taking photographs in Death Valley on a day when the temperature is $58.0^{\circ} \mathrm{C}$, Bill Hiker finds that a certain voltage applied to a copper wire produces a current of 1.000 A. Bill then travels to Antarctica and applies the
same voltage to the same wire. What current does he register there if the temperature is $-88.0^{\circ} \mathrm{C}$ ? Assume that no change occurs in the wire's shape and size.
29. A certain lightbulb has a tungsten filament with a resistance of $19.0 \Omega$ when cold and $140 \Omega$ when hot. Assume that the resistivity of tungsten varies linearly with temperature even over the large temperature range involved here, and find the temperature of the hot filament. Assume the initial temperature is $20.0^{\circ} \mathrm{C}$.
30. A carbon wire and a Nichrome wire are connected in series, so that the same current exists in both wires. If the combination has a resistance of $10.0 \mathrm{k} \Omega$ at $0^{\circ} \mathrm{C}$, what is the resistance of each wire at $0^{\circ} \mathrm{C}$ so that the resistance of the combination does not change with temperature? The total or equivalent resistance of resistors in series is the sum of their individual resistances.
31. An aluminum wire with a diameter of 0.100 mm has a uniform electric field of $0.200 \mathrm{~V} / \mathrm{m}$ imposed along its entire length. The temperature of the wire is $50.0^{\circ} \mathrm{C}$. Assume one free electron per atom. (a) Use the information in Table 27.1 and determine the resistivity. (b) What is the current density in the wire? (c) What is the total current in the wire? (d) What is the drift speed of the conduction electrons? (e) What potential difference must exist between the ends of a $2.00-\mathrm{m}$ length of the wire to produce the stated electric field?
32. Review problem. An aluminum rod has a resistance of $1.234 \Omega$ at $20.0^{\circ} \mathrm{C}$. Calculate the resistance of the $\operatorname{rod}$ at $120^{\circ} \mathrm{C}$ by accounting for the changes in both the resistivity and the dimensions of the rod.
33. What is the fractional change in the resistance of an iron filament when its temperature changes from $25.0^{\circ} \mathrm{C}$ to $50.0^{\circ} \mathrm{C}$ ?
34. The resistance of a platinum wire is to be calibrated for low-temperature measurements. A platinum wire with resistance $1.00 \Omega$ at $20.0^{\circ} \mathrm{C}$ is immersed in liquid nitrogen at $77 \mathrm{~K}\left(-196^{\circ} \mathrm{C}\right)$. If the temperature response of the platinum wire is linear, what is the expected resistance of the platinum wire at $-196^{\circ} \mathrm{C}$ ? $\left(\alpha_{\text {platinum }}=3.92 \times 10^{-3} /{ }^{\circ} \mathrm{C}\right)$
35. The temperature of a sample of tungsten is raised while a sample of copper is maintained at $20.0^{\circ} \mathrm{C}$. At what temperature will the resistivity of the tungsten be four times that of the copper?

## Section 27.6 Electrical Power

36. A toaster is rated at 600 W when connected to a $120-\mathrm{V}$ source. What current does the toaster carry, and what is its resistance?
37. A Van de Graaff generator (see Figure 25.29) is operating so that the potential difference between the high-voltage electrode $B$ and the charging needles at $A$ is 15.0 kV . Calculate the power required to drive the belt against electrical forces at an instant when the effective current delivered to the high-voltage electrode is $500 \mu \mathrm{~A}$.
38. In a hydroelectric installation, a turbine delivers 1500 hp to a generator, which in turn transfers $80.0 \%$ of the mechanical energy out by electrical transmission. Under
these conditions, what current does the generator deliver at a terminal potential difference of 2000 V ?
39. What is the required resistance of an immersion heater that increases the temperature of 1.50 kg of water from $10.0^{\circ} \mathrm{C}$ to $50.0^{\circ} \mathrm{C}$ in 10.0 min while operating at 110 V ?
40. One rechargeable battery of mass 15.0 g delivers to a CD player an average current of 18.0 mA at 1.60 V for 2.40 h before the battery needs to be recharged. The recharger maintains a potential difference of 2.30 V across the battery and delivers a charging current of 13.5 mA for 4.20 h . (a) What is the efficiency of the battery as an energy storage device? (b) How much internal energy is produced in the battery during one charge-discharge cycle? (b) If the battery is surrounded by ideal thermal insulation and has an overall effective specific heat of $975 \mathrm{~J} / \mathrm{kg}^{\circ} \mathrm{C}$, by how much will its temperature increase during the cycle?
41. Suppose that a voltage surge produces 140 V for a moment. By what percentage does the power output of a $120-\mathrm{V}, 100-\mathrm{W}$ lightbulb increase? Assume that its resistance does not change.
42. A $500-\mathrm{W}$ heating coil designed to operate from 110 V is made of Nichrome wire 0.500 mm in diameter. (a) Assuming that the resistivity of the Nichrome remains constant at its $20.0^{\circ} \mathrm{C}$ value, find the length of wire used. (b) What If? Now consider the variation of resistivity with temperature. What power will the coil of part (a) actually deliver when it is heated to $1200^{\circ} \mathrm{C}$ ?
43. A coil of Nichrome wire is 25.0 m long. The wire has a diameter of 0.400 mm and is at $20.0^{\circ} \mathrm{C}$. If it carries a current of 0.500 A , what are (a) the magnitude of the electric field in the wire, and (b) the power delivered to it? (c) What If? If the temperature is increased to $340^{\circ} \mathrm{C}$ and the voltage across the wire remains constant, what is the power delivered?
44. Batteries are rated in terms of ampere-hours $(A \cdot h)$. For example, a battery that can produce a current of 2.00 A for 3.00 h is rated at $6.00 \mathrm{~A} \cdot \mathrm{~h}$. (a) What is the total energy, in kilowatt-hours, stored in a $12.0-\mathrm{V}$ battery rated at $55.0 \mathrm{~A} \cdot \mathrm{~h}$ ? (b) At $\$ 0.0600$ per kilowatt-hour, what is the value of the electricity produced by this battery?
45. A $10.0-\mathrm{V}$ battery is connected to a $120-\Omega$ resistor. Ignoring the internal resistance of the battery, calculate the power delivered to the resistor.
46. Residential building codes typically require the use of 12 -gauge copper wire (diameter 0.2053 cm ) for wiring receptacles. Such circuits carry currents as large as 20 A . A wire of smaller diameter (with a higher gauge number) could carry this much current, but the wire could rise to a high temperature and cause a fire. (a) Calculate the rate at which internal energy is produced in 1.00 m of 12-gauge copper wire carrying a current of 20.0 A . (b) What If? Repeat the calculation for an aluminum wire. Would a 12 -gauge aluminum wire be as safe as a copper wire?
47. An $11.0-\mathrm{W}$ energy-efficient fluorescent lamp is designed to produce the same illumination as a conventional $40.0-\mathrm{W}$ incandescent lightbulb. How much money does the user of the energy-efficient lamp save during 100 hours of use? Assume a cost of $\$ 0.0800 / \mathrm{kWh}$ for energy from the power company.
48. We estimate that 270 million plug-in electric clocks are in the United States, approximately one clock for each person. The clocks convert energy at the average rate 2.50 W. To supply this energy, how many metric tons of coal are burned per hour in coal-fired electric generating plants that are, on average, $25.0 \%$ efficient? The heat of combustion for coal is $33.0 \mathrm{MJ} / \mathrm{kg}$.
49. Compute the cost per day of operating a lamp that draws a current of 1.70 A from a $110-\mathrm{V}$ line. Assume the cost of energy from the power company is $\$ 0.0600 / \mathrm{kWh}$.
50. Review problem. The heating element of a coffee maker operates at 120 V and carries a current of 2.00 A . Assuming that the water absorbs all of the energy delivered to the resistor, calculate how long it takes to raise the temperature of 0.500 kg of water from room temperature $\left(23.0^{\circ} \mathrm{C}\right)$ to the boiling point.
51. A certain toaster has a heating element made of Nichrome wire. When the toaster is first connected to a $120-\mathrm{V}$ source (and the wire is at a temperature of $20.0^{\circ} \mathrm{C}$ ), the initial current is 1.80 A . However, the current begins to decrease as the heating element warms up. When the toaster reaches its final operating temperature, the current drops to 1.53 A . (a) Find the power delivered to the toaster when it is at its operating temperature. (b) What is the final temperature of the heating element?
52. The cost of electricity varies widely through the United States; $\$ 0.120 / \mathrm{kWh}$ is one typical value. At this unit price, calculate the cost of (a) leaving a $40.0-\mathrm{W}$ porch light on for two weeks while you are on vacation, (b) making a piece of dark toast in 3.00 min with a $970-\mathrm{W}$ toaster, and (c) drying a load of clothes in 40.0 min in a $5200-\mathrm{W}$ dryer.
53. Make an order-of-magnitude estimate of the cost of one person's routine use of a hair dryer for 1 yr . If you do not use a blow dryer yourself, observe or interview someone who does. State the quantities you estimate and their values.

## Additional Problems

54. One lightbulb is marked ' 25 W 120 V ,' and another ' 100 W 120 V '; this means that each bulb has its respective power delivered to it when plugged into a constant $120-\mathrm{V}$ potential difference. (a) Find the resistance of each bulb. (b) How long does it take for 1.00 C to pass through the dim bulb? Is the charge different in any way upon its exit from the bulb versus its entry? (c) How long does it take for 1.00 J to pass through the dim bulb? By what mechanisms does this energy enter and exit the bulb? (d) Find how much it costs to run the dim bulb continuously for 30.0 days if the electric company sells its product at $\$ 0.0700$ per kWh . What product does the electric company sell? What is its price for one SI unit of this quantity?
55. A charge $Q$ is placed on a capacitor of capacitance $C$. The capacitor is connected into the circuit shown in Figure P27.55, with an open switch, a resistor, and an initially uncharged capacitor of capacitance $3 C$. The switch is then closed and the circuit comes to equilibrium. In terms of $Q$ and $C$, find (a) the final potential difference between the plates of each capacitor, (b) the charge on each capacitor,
and (c) the final energy stored in each capacitor. (d) Find the internal energy appearing in the resistor.


Figure P27.55
56. A high-voltage transmission line with a diameter of 2.00 cm and a length of 200 km carries a steady current of 1000 A . If the conductor is copper wire with a free charge density of $8.49 \times 10^{28}$ electrons $/ \mathrm{m}^{3}$, how long does it take one electron to travel the full length of the line?
57. A more general definition of the temperature coefficient of resistivity is

$$
\alpha=\frac{1}{\rho} \frac{d \rho}{d T}
$$

where $\rho$ is the resistivity at temperature T. (a) Assuming that $\alpha$ is constant, show that

$$
\rho=\rho_{0} e^{\alpha\left(T-T_{0}\right)}
$$

where $\rho_{0}$ is the resistivity at temperature $T_{0}$. (b) Using the series expansion $e^{x} \approx 1+x$ for $x \ll 1$, show that the resistivity is given approximately by the expression $\rho=\rho_{0}\left[1+\alpha\left(T-T_{0}\right)\right]$ for $\alpha\left(T-T_{0}\right) \ll 1$.
58. A high-voltage transmission line carries 1000 A starting at 700 kV for a distance of 100 mi . If the resistance in the wire is $0.500 \Omega / \mathrm{mi}$, what is the power loss due to resistive losses?
9. 20 An experiment is conducted to measure the electrical resistivity of Nichrome in the form of wires with different lengths and cross-sectional areas. For one set of measurements, a student uses 30 -gauge wire, which has a cross-sectional area of $7.30 \times 10^{-8} \mathrm{~m}^{2}$. The student measures the potential difference across the wire and the current in the wire with a voltmeter and an ammeter, respectively. For each of the measurements given in the table taken on wires of three different lengths, calculate the resistance of the wires and the corresponding values of the resistivity. What is the average value of the resistivity, and how does this value compare with the value given in Table 27.1?

| $\boldsymbol{L}(\mathbf{m})$ | $\boldsymbol{\Delta} \boldsymbol{V}(\mathbf{V})$ | $\boldsymbol{I}(\mathbf{A})$ | $\boldsymbol{R}(\boldsymbol{\Omega})$ | $\boldsymbol{\rho}(\boldsymbol{\Omega} \cdot \mathbf{m})$ |
| :--- | :--- | :--- | :--- | :--- |
| 0.540 | 5.22 | 0.500 |  |  |
| 1.028 | 5.82 | 0.276 |  |  |
| 1.543 | 5.94 | 0.187 |  |  |

60. An electric utility company supplies a customer's house from the main power lines ( 120 V ) with two copper wires, each of which is 50.0 m long and has a resistance of $0.108 \Omega$ per 300 m . (a) Find the voltage at the customer's
house for a load current of 110 A . For this load current, find (b) the power the customer is receiving and (c) the electric power lost in the copper wires.
61. A straight cylindrical wire lying along the $x$ axis has a length of 0.500 m and a diameter of 0.200 mm . It is made of a material that obeys Ohm's law with a resistivity of $\rho=$ $4.00 \times 10^{-8} \Omega \cdot \mathrm{~m}$. Assume that a potential of 4.00 V is maintained at $x=0$, and that $V=0$ at $x=0.500 \mathrm{~m}$. Find (a) the electric field $\mathbf{E}$ in the wire, (b) the resistance of the wire, (c) the electric current in the wire, and (d) the current density $\mathbf{J}$ in the wire. Express vectors in vector notation. (e) Show that $\mathbf{E}=\rho \mathbf{J}$.
62. A straight cylindrical wire lying along the $x$ axis has a length $L$ and a diameter $d$. It is made of a material that obeys Ohm's law with a resistivity $\rho$. Assume that potential $V$ is maintained at $x=0$, and that the potential is zero at $x=L$. In terms of $L, d, V, \rho$, and physical constants, derive expressions for (a) the electric field in the wire, (b) the resistance of the wire, (c) the electric current in the wire, and (d) the current density in the wire. Express vectors in vector notation. (e) Prove that $\mathbf{E}=\rho \mathbf{J}$.
63. The potential difference across the filament of a lamp is maintained at a constant level while equilibrium temperature is being reached. It is observed that the steady-state current in the lamp is only one tenth of the current drawn by the lamp when it is first turned on. If the temperature coefficient of resistivity for the lamp at $20.0^{\circ} \mathrm{C}$ is $0.00450\left({ }^{\circ} \mathrm{C}\right)^{-1}$, and if the resistance increases linearly with increasing temperature, what is the final operating temperature of the filament?
64. The current in a resistor decreases by 3.00 A when the voltage applied across the resistor decreases from 12.0 V to 6.00 V . Find the resistance of the resistor.

65 . An electric car is designed to run off a bank of $12.0-\mathrm{V}$ batteries with total energy storage of $2.00 \times 10^{7} \mathrm{~J}$. (a) If the electric motor draws 8.00 kW , what is the current delivered to the motor? (b) If the electric motor draws 8.00 kW as the car moves at a steady speed of $20.0 \mathrm{~m} / \mathrm{s}$, how far will the car travel before it is "out of juice"?
66. Review problem. When a straight wire is heated, its resistance is given by $R=R_{0}\left[1+\alpha\left(T-T_{0}\right)\right]$ according to Equation 27.21, where $\alpha$ is the temperature coefficient of resistivity. (a) Show that a more precise result, one that includes the fact that the length and area of the wire change when heated, is

$$
R=\frac{R_{0}\left[1+\alpha\left(T-T_{0}\right)\right]\left[1+\alpha^{\prime}\left(T-T_{0}\right)\right]}{\left[1+2 \alpha^{\prime}\left(T-T_{0}\right)\right]}
$$

where $\alpha^{\prime}$ is the coefficient of linear expansion (see Chapter 19). (b) Compare these two results for a $2.00-\mathrm{m}$-long copper wire of radius 0.100 mm , first at $20.0^{\circ} \mathrm{C}$ and then heated to $100.0^{\circ} \mathrm{C}$.
67. The temperature coefficients of resistivity in Table 27.1 were determined at a temperature of $20^{\circ} \mathrm{C}$. What would they be at $0^{\circ} \mathrm{C}$ ? Note that the temperature coefficient of resistivity at $20^{\circ} \mathrm{C}$ satisfies $\rho=\rho_{0}\left[1+\alpha\left(T-T_{0}\right)\right]$, where $\rho_{0}$ is the resistivity of the material at $T_{0}=20^{\circ} \mathrm{C}$. The temperature
coefficient of resistivity $\alpha^{\prime}$ at $0^{\circ} \mathrm{C}$ must satisfy the expression $\rho=\rho_{0}^{\prime}\left[1+\alpha^{\prime} T\right]$, where $\rho_{0}^{\prime}$ is the resistivity of the material at $0^{\circ} \mathrm{C}$.
68. An oceanographer is studying how the ion concentration in sea water depends on depth. She does this by lowering into the water a pair of concentric metallic cylinders (Fig. P27.68) at the end of a cable and taking data to determine the resistance between these electrodes as a function of depth. The water between the two cylinders forms a cylindrical shell of inner radius $r_{a}$, outer radius $r_{b}$, and length $L$ much larger than $r_{b}$. The scientist applies a potential difference $\Delta V$ between the inner and outer surfaces, producing an outward radial current $I$. Let $\rho$ represent the resistivity of the water. (a) Find the resistance of the water between the cylinders in terms of $L$, $\rho, r_{a}$, and $r_{b}$. (b) Express the resistivity of the water in terms of the measured quantities $L, r_{a}, r_{b}, \Delta V$, and $I$.


Figure P27.68
69. In a certain stereo system, each speaker has a resistance of $4.00 \Omega$. The system is rated at 60.0 W in each channel, and each speaker circuit includes a fuse rated 4.00 A . Is this system adequately protected against overload? Explain your reasoning.
70. A close analogy exists between the flow of energy by heat because of a temperature difference (see Section 20.7) and the flow of electric charge because of a potential difference. The energy $d Q$ and the electric charge $d q$ can both be transported by free electrons in the conducting material. Consequently, a good electrical conductor is usually a good thermal conductor as well. Consider a thin conducting slab of thickness $d x$, area $A$, and electrical conductivity $\sigma$, with a potential difference $d V$ between opposite faces. Show that the current $I=d q / d t$ is given by the equation on the left below:

| Charge Conduction | Thermal Conduc <br> (Eq. 20.14) |
| :--- | :--- |
| $\frac{d q}{d t}=\sigma A\left\|\frac{d V}{d x}\right\|$ | $\frac{d Q}{d t}=k A\left\|\frac{d T}{d x}\right\|$ |

In the analogous thermal conduction equation on the right, the rate of energy flow $d Q / d t$ (in SI units of joules per second) is due to a temperature gradient $d T / d x$, in a material of thermal conductivity $k$. State analogous rules relating the direction of the electric current to the change in potential, and relating the direction of energy flow to the change in temperature.
71. Material with uniform resistivity $\rho$ is formed into a wedge as shown in Figure P27.71. Show that the resistance between face $A$ and face $B$ of this wedge is

$$
R=\rho \frac{L}{w\left(y_{2}-y_{1}\right)} \ln \left(\frac{y_{2}}{y_{1}}\right)
$$


72. A material of resistivity $\rho$ is formed into the shape of a truncated cone of altitude $h$ as shown in Figure P27.72. The bottom end has radius $b$, and the top end has radius $a$. Assume that the current is distributed uniformly over any circular cross section of the cone, so that the current density does not depend on radial position. (The current density does vary with position along the axis of the cone.) Show that the resistance between the two ends is described by the expression

$$
R=\frac{\rho}{\pi}\left(\frac{h}{a b}\right)
$$



Figure P27.72
73. The dielectric material between the plates of a parallelplate capacitor always has some nonzero conductivity $\sigma$. Let $A$ represent the area of each plate and $d$ the distance between them. Let $\kappa$ represent the dielectric constant of the material. (a) Show that the resistance $R$ and the capacitance $C$ of the capacitor are related by

$$
R C=\frac{\kappa \epsilon_{0}}{\sigma}
$$

(b) Find the resistance between the plates of a $14.0-\mathrm{nF}$ capacitor with a fused quartz dielectric.
74. $\square$ The current-voltage characteristic curve for a semiconductor diode as a function of temperature $T$ is given by the equation

$$
I=I_{0}\left(e^{e \Delta V / k_{\mathrm{B}} T}-1\right)
$$

Here the first symbol $e$ represents Euler's number, the base of natural logarithms. The second $e$ is the charge on the electron. The $k_{\mathrm{B}}$ stands for Boltzmann's constant, and $T$ is the absolute temperature. Set up a spreadsheet to calculate $I$ and $R=\Delta V / I$ for $\Delta V=0.400 \mathrm{~V}$ to 0.600 V in increments of 0.005 V . Assume $I_{0}=1.00 \mathrm{nA}$. Plot $R$ versus $\Delta V$ for $T=280 \mathrm{~K}, 300 \mathrm{~K}$, and 320 K .
75. Review problem. A parallel-plate capacitor consists of square plates of edge length $\ell$ that are separated by a distance $d$, where $d \ll \ell$. A potential difference $\Delta V$ is maintained between the plates. A material of dielectric constant $\kappa$ fills half of the space between the plates. The dielectric slab is now withdrawn from the capacitor, as shown in Figure P27.75. (a) Find the capacitance when the left edge of the dielectric is at a distance $x$ from the center of the capacitor. (b) If the dielectric is removed at a constant speed $v$, what is the current in the circuit as the dielectric is being withdrawn?


## Answers to Quick Quizzes

$27.1 \mathrm{~d}, \mathrm{~b}=\mathrm{c}$, a . The current in part ( d ) is equivalent to two positive charges moving to the left. Parts (b) and (c) each represent four positive charges moving in the same direction because negative charges moving to the left are equivalent to positive charges moving to the right. The current in part (a) is equivalent to five positive charges moving to the right.
27.2 (b). The currents in the two paths add numerically to equal the current coming into the junction, without regard for the directions of the two wires coming out of
the junction. This is indicative of scalar addition. Even though we can assign a direction to a current, it is not a vector. This suggests a deeper meaning for vectors besides that of a quantity with magnitude and direction.
27.3 (a). The current in each section of the wire is the same even though the wire constricts. As the cross-sectional area $A$ decreases, the drift velocity must increase in order for the constant current to be maintained, in accordance with Equation 27.4. As $A$ decreases, Equation 27.11 tells us that $R$ increases.
27.4 (b). The doubling of the radius causes the area $A$ to be four times as large, so Equation 27.11 tells us that the resistance decreases.
27.5 (b). The slope of the tangent to the graph line at a point is the reciprocal of the resistance at that point. Because the slope is increasing, the resistance is decreasing.
27.6 (a). When the filament is at room temperature, its resistance is low, and hence the current is relatively large. As the filament warms up, its resistance increases, and the current decreases. Older lightbulbs often fail just as they are turned on because this large initial current "spike" produces rapid temperature increase and mechanical stress on the filament, causing it to break.
27.7 (c). Because the potential difference $\Delta V$ is the same across the two bulbs and because the power delivered to a conductor is $\mathscr{P}=I \Delta V$, the $60-\mathrm{W}$ bulb, with its higher power rating, must carry the greater current. The $30-\mathrm{W}$ bulb has the higher resistance because it draws less current at the same potential difference.
27.8 $I_{a}=I_{b}>I_{c}=I_{d}>I_{e}=I_{f}$. The current $I_{a}$ leaves the positive terminal of the battery and then splits to flow through the two bulbs; thus, $I_{a}=I_{c}+I_{e}$. From Quick Quiz 27.7, we know that the current in the $60-\mathrm{W}$ bulb is greater than that in the $30-\mathrm{W}$ bulb. Because charge does not build up in the bulbs, we know that the same amount of charge flowing into a bulb from the left must flow out on the right; consequently, $I_{c}=I_{d}$ and $I_{e}=I_{f}$. The two currents leaving the bulbs recombine to form the current back into the battery, $I_{f}+I_{d}=I_{b}$.

